IDS 702: Module 6.1

The potential outcomes framework and causal estimands

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> We do not have knowledge of a thing until we have grasped its why, that is to say, its cause.

-- Aristotle, Physics

- Over the next few modules, we will discuss causal inference, specifically, on measuring the effects of causes.
- For now, we will simply lay the foundations for causal inference.
- We will get more into the actual methods later.

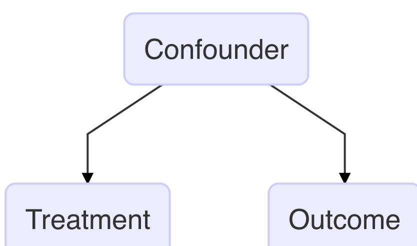
Association vs. causation

- In the models we have covered so far, our focus has been on inferring associations using samples drawn from our population of interest.
- For example, we have been asking questions such as, do people who receive job training tend to earn more wages than people who do not?
- Causal inference goes further as we try to infer aspects of the actual data generating process, that is, causation.
- For example, does receiving job training actually cause one to earn more wage than they would have without the training?
- The additional information needed to move from association to causation is often provided by causal assumptions (often untestable).
- Note: in most cases, association does not imply causation!

Confounding

- Why is it that association does not often imply causation? confounding variables or confounders!
- Causal relationship

- Confounding



Examples of confounding

- Ice cream consumption and number of people who drowned.
  Confounder: temperature; people tend to consume more ice cream and also swim more when it is hot.
- Medical treatment and patient outcome.
  Confounders: age, sex, other complications
- Education and income.
  Confounder: socio-economic status of family
- An extreme example of confounding is Simpson's paradox: where a confounder reverses the sign of the correlation between treatment and outcome

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Simpson's paradox

- Example: kidney stone treatment (Charig et al., BMJ, 1986).
    - Compare the success rates of two treatments for kidney stones
    - Treatment A: open surgery. Treatment B: small puncture

	Treatment A	Treatment B
Small stones	**93%** (81/87)	87% (234/270)
Large stones	**73%** (192/263)	69% (55/80)
Both	78% (273/350)	**83%** (289/350)

- Overall treatmnent B has a higher success rate but treatment A actually has higher success rates given the type of stones.
- What is the confounder here? Severity of the case/type of stones.

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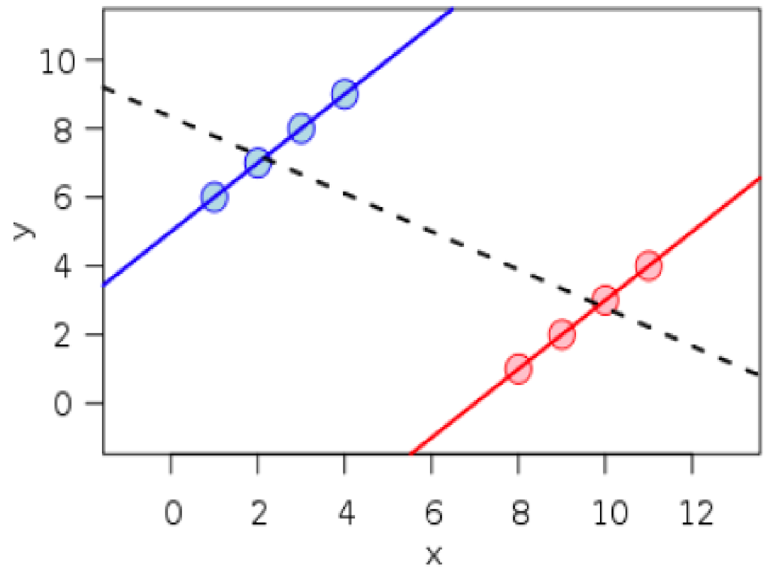
Simpson's paradox or Yule-Simpson effect

- Simpson's paradox: a trend appears in different groups of data but disappears or reverses when these groups are combined.



- Mathematically, it is about conditioning.
- Another well-known example is the Berkeley admission gender bias (Bickel et al., Science, 1976).

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General notation

- W: Treatment (e.g. job training); we will focus on binary treatments.
- Y: Outcome (e.g. annual wages).
- X: Observed predictors or confounders (e.g. age, education, etc).
- U: Unobserved predictors or confounders.

- Examples of causal questions:
  - Causal effect of exposure to a disease.
  - Comparative effectiveness research such as in clinical trials: whether one drug or medical procedure is better than the other.
  - Program evaluation in economics and policy.

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Potential outcomes framework

Potential outcomes framework

- The potential outcomes framework or counterfactual framework or Rubin Causal Model (RCM) is arguably the most widely used framework across many disciplines, e.g., medicine, health care, policy, social sciences.
- Under this framework, causal inference is viewed as a problem of missing data with explicit mathematical modeling of the assignment mechanism as a process for revealing the observed data.
- Rooted in the statistical work on randomized experiments by Fisher (1918, 1925) and Neyman (1923), as extended by Rubin (1974, 1976, 1977, 1978, 1990).

Potential outcomes framework

- For a binary treatment, each individual gets exactly one of the two options, and we observe the corresponding response for that.
- Conceptually, under the potential outcomes framework, we think about what each individual's response should have been had they gotten the other treatment option instead of the one they actually got.
- The individual causal effect then is the difference between the two "potential" outcomes, only one of which is observed.
- Clearly, we never observe the two potential outcomes for any individual, making it natural to think of this as a missing data problem.

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Potential outcomes framework

- No causation without manipulation - "cause" must be (hypothetically speaking) something we can manipulate. e.g., intervention, action, treatment.
- That is, gender, time and age are not well defined “causes" under the RCM.
- Three integral components of the potential outcomes framework:
  - potential outcomes corresponding to the various levels of a treatment.
  - assignment mechanisms, that is, the treatment indicator for all observations.
  - a model for the science (the potential outcomes and covariates).

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Potential outcomes framework: basic concepts

- Unit: The person, place, or thing upon which a treatment will operate, at a particular time (note: a single person, place, or thing at two different times comprises two different units).

- Treatment: An intervention, the effects of which (on some particular measurement of the units) the investigator wishes to assess relative to no intervention (i.e., the control).

- Potential Outcomes: The values of a unit's measurement of interest after (a) application of the treatment and (b) non-application of the treatment (i.e., under control).

- Causal Effect: For each unit, the comparison of the potential outcome under treatment and the potential outcome under control.

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Causal effects

- For a single unit, let $Y(0)$ denote the outcome given the control treatment and $Y(1)$, the outcome given the active treatment.
- For example, suppose Y denotes a score (level of severity) for headache, then for a single unit, we could have

Raw scores

Unit	Initial headache	Potential outcomes		Causal effect
	X	Y(asp)	Y(not)	Y(asp) - Y(not)
you	80	25	75	-50

Gain scores

Unit	Initial headache	Potential outcomes		Causal effect
	X	Y(asp) - X	Y(not) - X	[Y(asp) - X] - [Y(not) - X]
you	80	-55	-5	-50

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The fundamental problem of causal inference

As mentioned before,

- The fundamental problem of causal inference: we can observe at most one of the potential outcomes for each unit.
- Causal inference under the potential outcome framework is essentially a missing data problem.
- To identify causal effects from observed data, under any mathematical framework, one must make assumptions (structural or/and stochastic)
- Since we can at most observe a single potential outcome, we must rely on multiple units (and a lot of assumptions) to make causal inferences.

Basic setup

- Target population: a well-defined population of individuals whose outcomes are going to be compared
- Data: a random sample of N units from a target population.
- A treatment with two levels: $w = 0, 1$.
- For each unit i, we observe
  - the binary treatment status $W_i \in \{0, 1\}$,
  - a vector of p predictors/covariates $X_i = (X_{i1}, \ldots, X_{ip})$, and
  - an outcome Y_i^{obs}.

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Basic setup

- For each unit i, there are two potential outcomes $(Y_i(0), Y_i(1))$.
- That is, the outcomes under the two values of the treatment, at most one of which is observed.
- Potential outcomes and assignments jointly determine the values of the observed outcomes

$$Y_i^{\text{obs}} \equiv Y_i(W_i) = W_i \cdot Y_i(1) + (1 - W_i) \cdot Y_i(0)$$

and the missing outcomes:

$$Y_i^{\text{mis}} \equiv Y_i(1 - W_i) = (1 - W_i) \cdot Y_i(1) + W_i \cdot Y_i(0)$$

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Causal estimands

- The average treatment effect (ATE):

$$\tau = \mathbb{E}[Y_i(1) - Y_i(0)].$$

- The average treatment effect for the treated (ATT):

$$\tau = \mathbb{E}[Y_i(1) - Y_i(0)|W_i = 1].$$

- The average treatment effect for the control (ATC):

$$\tau = \mathbb{E}[Y_i(1) - Y_i(0)|W_i = 0].$$

- For binary outcomes, causal odds ratio (OR) or risk ratio (RR)::

$$\tau = \frac{\mathbb{P}\mathrm{r}[Y_i(1) = 1]/\mathbb{P}\mathrm{r}[Y_i(1) = 0]}{\mathbb{P}\mathrm{r}[Y_i(0) = 1]/\mathbb{P}\mathrm{r}[Y_i(0) = 0]}.$$

- Obviously these estimands are not identifiable without further assumptions.

- We will start to explore those soon.

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Example

	Potential Outcomes			Observed Data		
	Y(0)	Y(1)		W	Y(0)	Y(1)
	13	14		1	?	14
	6	0		0	6	?
	4	1		0	4	?
	5	2		0	5	?
	6	3		0	6	?
	6	1		0	6	?
	8	10		1	?	10
	8	9		1	?	9
True averages	7	5	Observed averages		5.4	11

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Acknowledgements

These slides contain materials adapted from courses taught by Dr. Fan Li.

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What's next?

Move on to the readings for the next module!

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