

IDS 702: MODULE 5.4

MULTIPLE IMPUTATION IN R

DR. OLANREWAJU MICHAEL AKANDE

ILLUSTRATION

- Simple example using data that come with the **MICE** package in R.
- Dataset from NHANES includes 25 cases measured on 4 variables.
- Only 13 cases with complete data.
- We will use multiple imputation to make completed datasets and do analyses.
- The four variables are
 1. age (age group: 20-39, 40-59, 60+)
 2. bmi (body mass index, in kg/m^2)
 3. hyp (hypertension status: no, yes)
 4. chl (total cholesterol, in mg/dL)

ILLUSTRATION

```
library(mice)
data(nhanes2)
dim(nhanes2)

## [1] 25  4

summary(nhanes2)

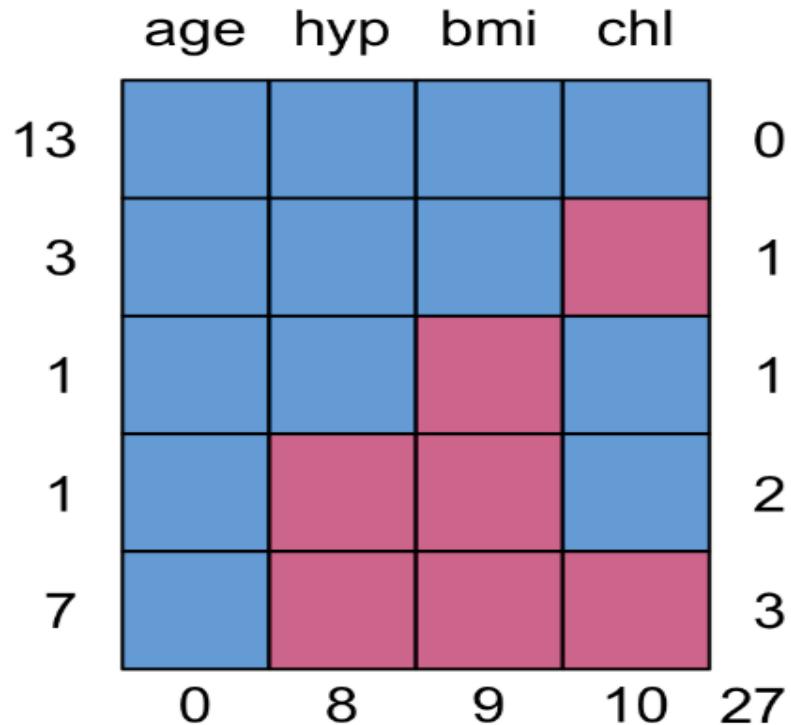
##      age          bmi          hyp          chl
## 20-39:12   Min.   :20.40   no   :13   Min.   :113.0
## 40-59: 7    1st Qu.:22.65  yes  : 4   1st Qu.:185.0
## 60-99: 6    Median :26.75  NA's: 8   Median :187.0
##                  Mean   :26.56               Mean   :191.4
##                  3rd Qu.:28.93              3rd Qu.:212.0
##                  Max.   :35.30               Max.   :284.0
##                  NA's   :9                 NA's   :10

str(nhanes2)

## 'data.frame': 25 obs. of 4 variables:
## $ age: Factor w/ 3 levels "20-39","40-59",...: 1 2 1 3 1 3 1 1 2 2 ...
## $ bmi: num NA 22.7 NA NA 20.4 NA 22.5 30.1 22 NA ...
## $ hyp: Factor w/ 2 levels "no","yes": NA 1 1 NA 1 NA 1 1 1 NA ...
## $ chl: num NA 187 187 NA 113 184 118 187 238 NA ...
```

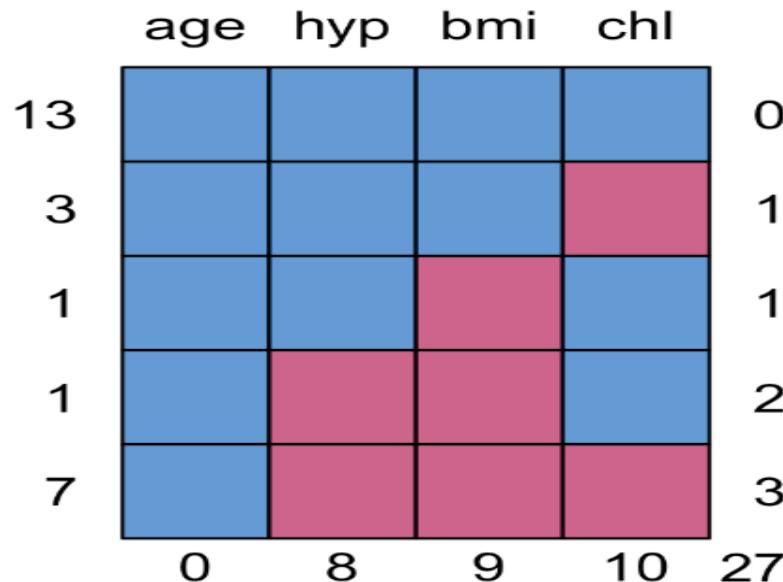
PATTERNS OF MISSING DATA

```
md.pattern(nhanes2)
```



5 patterns observed from $2^3 = 8$ possible patterns

PATTERNS OF MISSING DATA



- **At the bottom:** total number of missing values by variables.
- **On the right:** number of variables missing in each pattern.
- **On the left:** number of cases for each pattern.

VISUALIZING PATTERNS OF MISSING DATA

```
library(VIM); library(lattice)
aggr(nhanes2,col=c("lightblue3","darkred"),numbers=TRUE,sortVars=TRUE,
      labels=names(nhanes2),cex.axis=.7,gap=3,
      ylab=c("Proportion missing","Missingness pattern"))

## 
## Variables sorted by number of missings:
## Variable Count
##     chl  0.40
##     bmi  0.36
##     hyp  0.32
##     age  0.00
```

VISUALIZING PATTERNS OF MISSING DATA

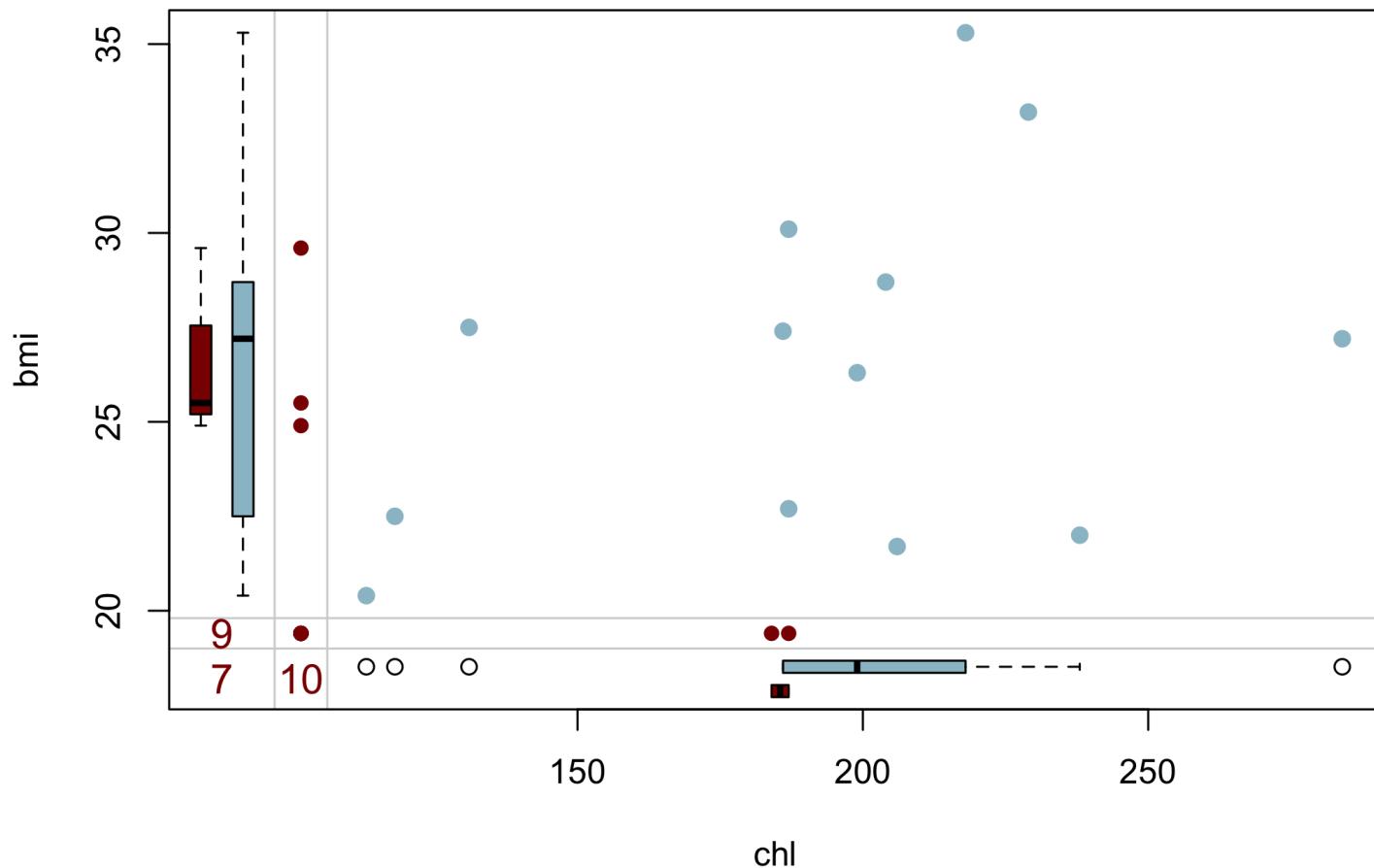
The **marginplot** function can be used to understand how missingness affects the distribution of values on other variables.

- Blue box plots summarize the distribution of observed data given the other variable is observed.
- Red box plots summarize the distribution of observed data given the other variable is missing.
- If data are MCAR, you expect the boxplots to be the same (hard to evaluate in this small sample)

Let's look at the margin plot for the two continuous variables `bmi` and `chl`.

VISUALIZING PATTERNS OF MISSING DATA

```
marginplot(nhanes2[,c("chl","bmi")], col=c("lightblue3","darkred"), cex.numbers=1.2, pch=19)
```



VISUALIZING PATTERNS OF MISSING DATA

- Interpretation of the numbers in red.
 - 9 = number of observations with missingness in `bmi`
 - 10 = number of observations with missingness in `chl`
 - 7 = number of observations with missingness in both `bmi` and `chl`.
- The scatterplot of blue points display the relationship between `bmi` and `chl` when they are both observed (13 cases).
- The red points indicate the amount of data used to generate the red boxplots.

MICE IN R

We will use the `mice` function to generate 10 imputed datasets. By default, `mice` uses

- `pmm`: Predictive mean matching for numeric data
- `logreg`: Logistic regression for factor data with 2 levels
- `polyreg`: Multinomial logistic regression for factor data with > 2 levels
- `polr`: Proportional odds model for factor data with > 2 ordered levels

MICE IN R

Other commonly used methods are

- **norm**: Bayesian linear regression
- **sample**: Random sample from observed values
- **cart**: Classification and regression trees
- **rf**: Random forest

Personally, I prefer to use **norm** instead of **pmm** for imputing numeric/continuous variables.

For the illustration,

```
nhanes2_imp <- mice(nhanes2,m=10,  
                      defaultMethod=c("norm","logreg","polyreg","polr"),  
                      print=F)
```

MICE IN R

```
methods(mice)
```

```
## Warning in .S3methods(generic.function, class, envir): function 'mice' appears
## not to be S3 generic; found functions that look like S3 methods

## [1] mice.impute.2l.bin      mice.impute.2l.lmer      mice.impute.2l.norm
## [4] mice.impute.2l.pan      mice.impute.2lonly.mean  mice.impute.2lonly.norm
## [7] mice.impute.2lonly.pmm   mice.impute.cart       mice.impute.jomoImpute
## [10] mice.impute.lda       mice.impute.logreg     mice.impute.logreg.boot
## [13] mice.impute.mean       mice.impute.midastouch  mice.impute.mnar.logreg
## [16] mice.impute.mnar.norm   mice.impute.norm       mice.impute.norm.boot
## [19] mice.impute.norm.nob    mice.impute.norm.predict mice.impute.panImpute
## [22] mice.impute.passive    mice.impute.pmm        mice.impute.polr
## [25] mice.impute.polyreg    mice.impute.quadratic  mice.impute.rf
## [28] mice.impute.ri         mice.impute.sample    mice.mids
## [31] mice.theme
## see '?methods' for accessing help and source code
```

PREDICTIVE MEAN MATCHING (PMM)

- Suppose y is subject to missing values while x is completely observed.
The basic idea for pmm is:
 - Using complete cases, regress y on x , obtaining $\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1)$;
 - Draw a new β^* from the "posterior distribution" of $\hat{\beta}$ (e.g, multivariate normal);
 - Using β^* , generate predicted values of y for all cases;
 - For each case with a missing y , identify set of donors with no missing values, who have predicted y values close to that of the case with missing data;
 - From among these cases, randomly select one and assign its observed value of y as the imputed value;
 - Repeat for all observations and imputation data sets.
- Pmm matches the distribution of the original observed variable, as imputed values are taken from the real data.

MICE IN R

Back to the `nhanes2_imp` object, first look at the original data

`nhanes2`

```
##      age   bmi   hyp chl
## 1  20-39    NA <NA>  NA
## 2  40-59 22.7   no 187
## 3  20-39    NA   no 187
## 4  60-99    NA <NA>  NA
## 5  20-39 20.4   no 113
## 6  60-99    NA <NA> 184
## 7  20-39 22.5   no 118
## 8  20-39 30.1   no 187
## 9  40-59 22.0   no 238
## 10 40-59    NA <NA>  NA
## 11 20-39    NA <NA>  NA
## 12 40-59    NA <NA>  NA
## 13 60-99 21.7   no 206
## 14 40-59 28.7  yes 204
## 15 20-39 29.6   no  NA
## 16 20-39    NA <NA>  NA
## 17 60-99 27.2  yes 284
## 18 40-59 26.3  yes 199
## 19 20-39 35.3   no 218
## 20 60-99 25.5  yes  NA
## 21 20-39    NA <NA>  NA
## 22 20-39 33.2   no 229
## 23 20-39 27.5   no 131
## 24 60-99 24.9   no  NA
## 25 40-59 27.4   no 186
```

MICE IN R

Look at the first imputed-dataset

```
d1 <- complete(nhanes2_imp, 1); d1
```

```
##      age      bmi hyp      chl
## 1  20-39 30.34098  no 224.2285
## 2  40-59 22.70000  no 187.0000
## 3  20-39 27.29756  no 187.0000
## 4  60-99 35.61975 yes 297.0469
## 5  20-39 20.40000  no 113.0000
## 6  60-99 18.16587 yes 184.0000
## 7  20-39 22.50000  no 118.0000
## 8  20-39 30.10000  no 187.0000
## 9  40-59 22.00000  no 238.0000
## 10 40-59 27.84743  no 210.5014
## 11 20-39 27.24996  no 146.9218
## 12 40-59 28.43579  no 226.0825
## 13 60-99 21.70000  no 206.0000
## 14 40-59 28.70000 yes 204.0000
## 15 20-39 29.60000  no 208.7224
## 16 20-39 36.73795  no 230.7358
## 17 60-99 27.20000 yes 284.0000
## 18 40-59 26.30000 yes 199.0000
## 19 20-39 35.30000  no 218.0000
## 20 60-99 25.50000 yes 257.7126
## 21 20-39 22.42268  no 127.4948
## 22 20-39 33.20000  no 229.0000
## 23 20-39 27.50000  no 131.0000
## 24 60-99 24.90000  no 283.3828
## 25 40-59 27.40000  no 186.0000
```

MICE IN R

Look at the last imputed-dataset

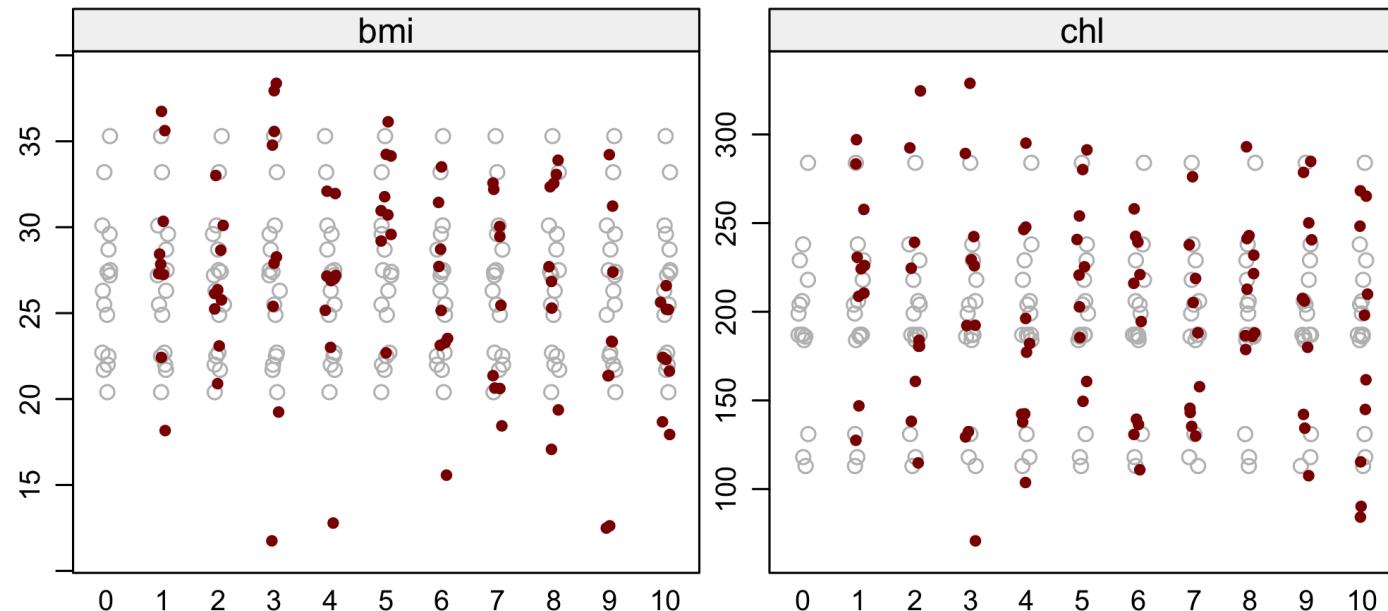
```
d10 <- complete(nhanes2_imp, 10); d10
```

```
##      age      bmi hyp      chl
## 1  20-39 26.59731  no 115.34762
## 2  40-59 22.70000  no 187.00000
## 3  20-39 22.42548  no 187.00000
## 4  60-99 25.20745 yes 268.22285
## 5  20-39 20.40000  no 113.00000
## 6  60-99 22.29470  no 184.00000
## 7  20-39 22.50000  no 118.00000
## 8  20-39 30.10000  no 187.00000
## 9  40-59 22.00000  no 238.00000
## 10 40-59 25.63055 yes 198.06595
## 11 20-39 17.93695  no  90.22238
## 12 40-59 21.62430 yes 209.92840
## 13 60-99 21.70000  no 206.00000
## 14 40-59 28.70000 yes 204.00000
## 15 20-39 29.60000  no 161.64022
## 16 20-39 18.67313  no  84.17346
## 17 60-99 27.20000 yes 284.00000
## 18 40-59 26.30000 yes 199.00000
## 19 20-39 35.30000  no 218.00000
## 20 60-99 25.50000 yes 265.20195
## 21 20-39 25.22149  no 144.88429
## 22 20-39 33.20000  no 229.00000
## 23 20-39 27.50000  no 131.00000
## 24 60-99 24.90000  no 248.27329
## 25 40-59 27.40000  no 186.00000
```

ILLUSTRATION

Let's plot imputed and observed values for continuous variables.

```
stripplot(nhanes2_imp, col=c("grey","darkred"), pch=c(1,20))
```

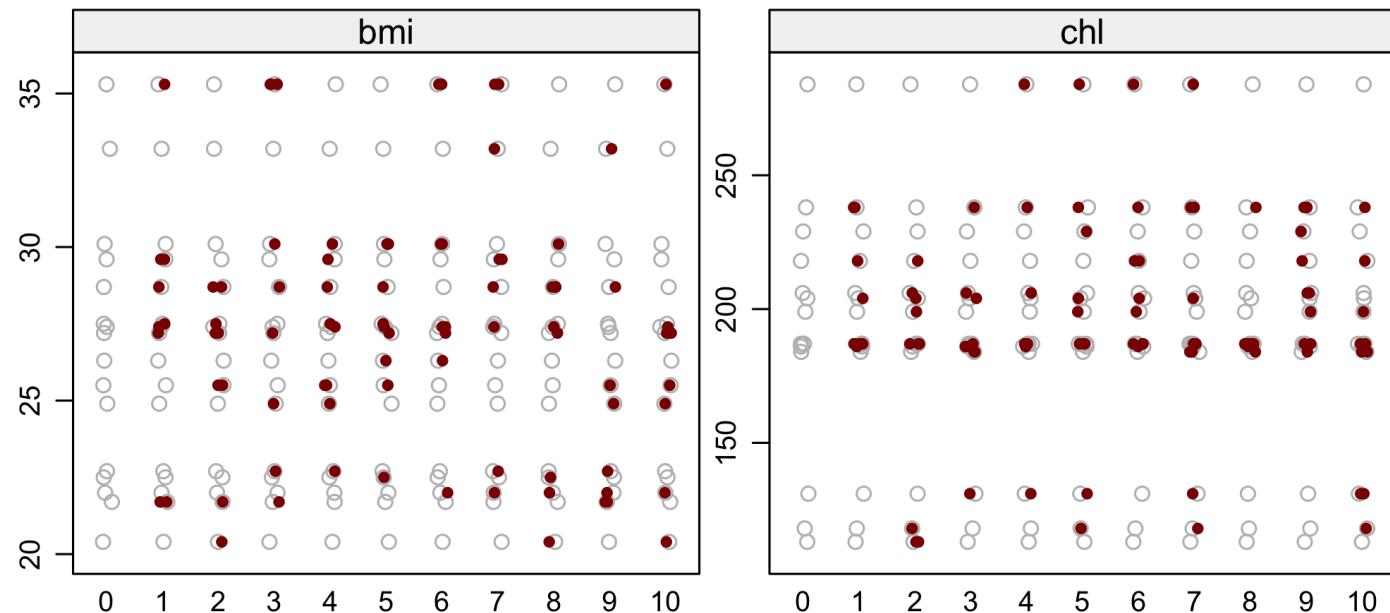


Grey dots are observed values and red dots are imputed values.

ILLUSTRATION

Let's see how this would change when we use `pmm` instead of `norm`.

```
nhanes2_imp2 <- mice(nhanes2,m=10,defaultMethod=c("pmm","logreg","polyreg","polr"),print=F  
stripplot(nhanes2_imp2, col=c("grey","darkred"),pch=c(1,20))
```

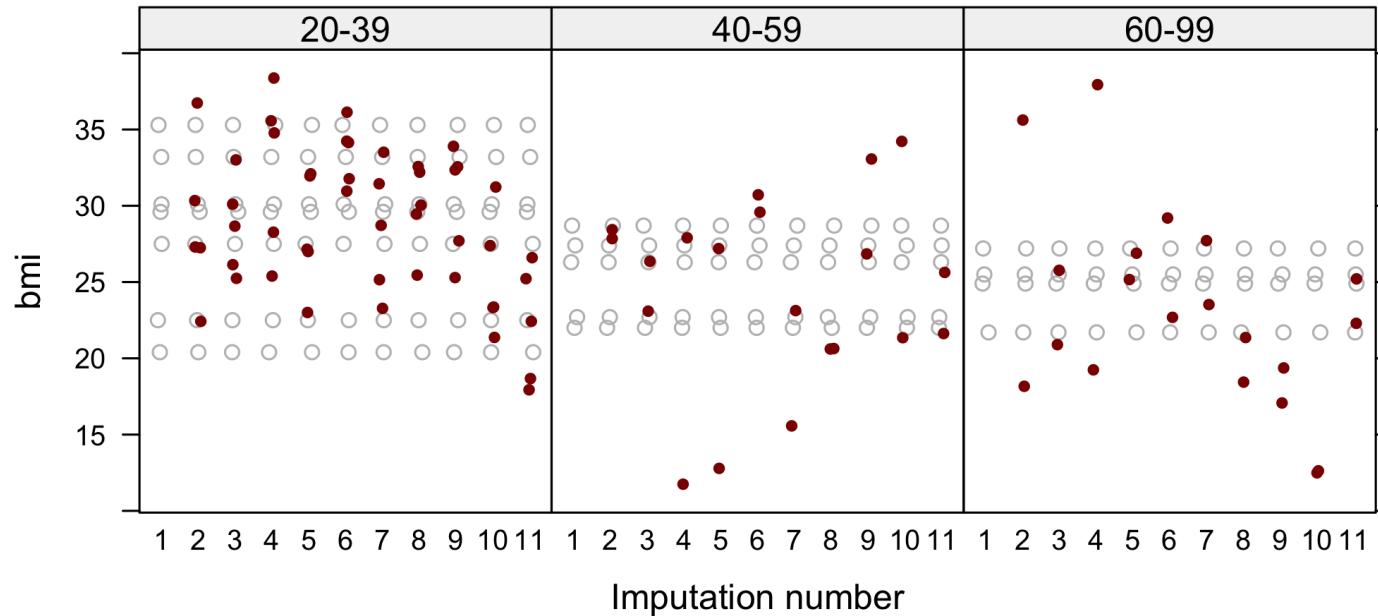


Easy to see that the distribution of the original observed data is preserved.

ILLUSTRATION

Also can do plots by values of categorical variable, say `bmi` by `age`. Let's look at the imputations using `norm`

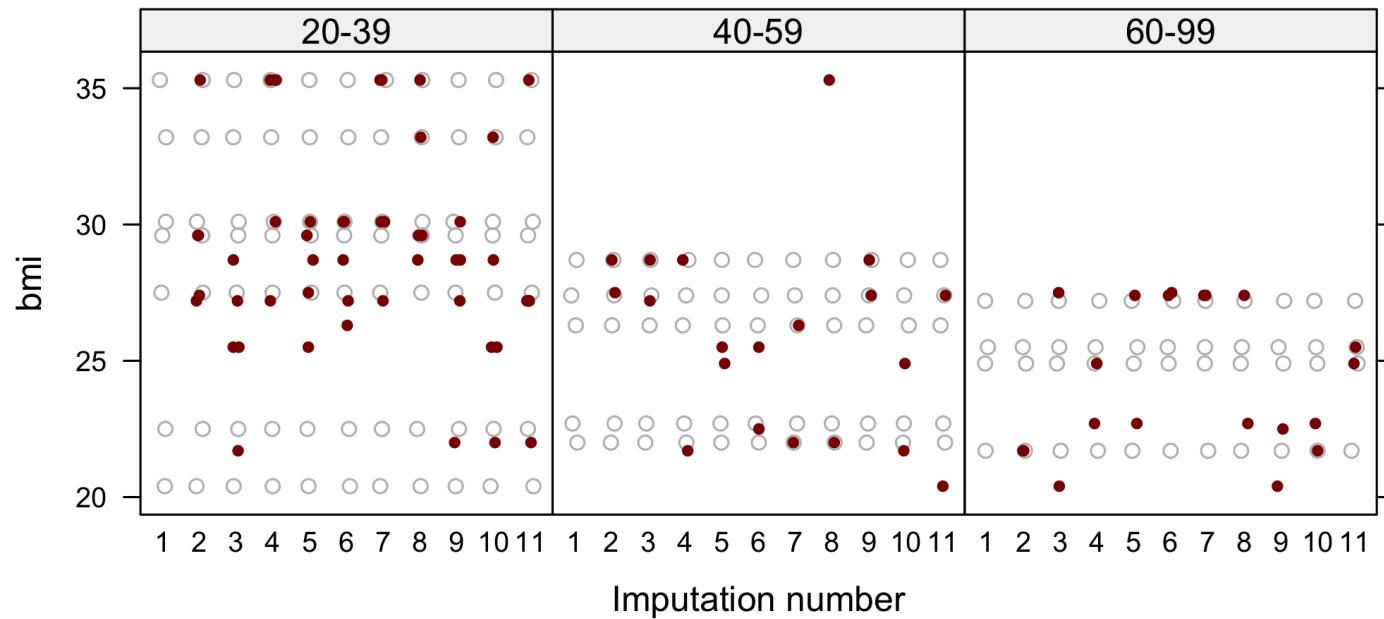
```
stripplot(nhanes2_imp, bmi~.imp|age, col=c("grey","darkred"),pch=c(1,20))
```



ILLUSTRATION

Using `pmm` instead of `norm`, we have:

```
stripplot(nhanes2_imp2, bmi~.imp|age, col=c("grey","darkred"),pch=c(1,20))
```

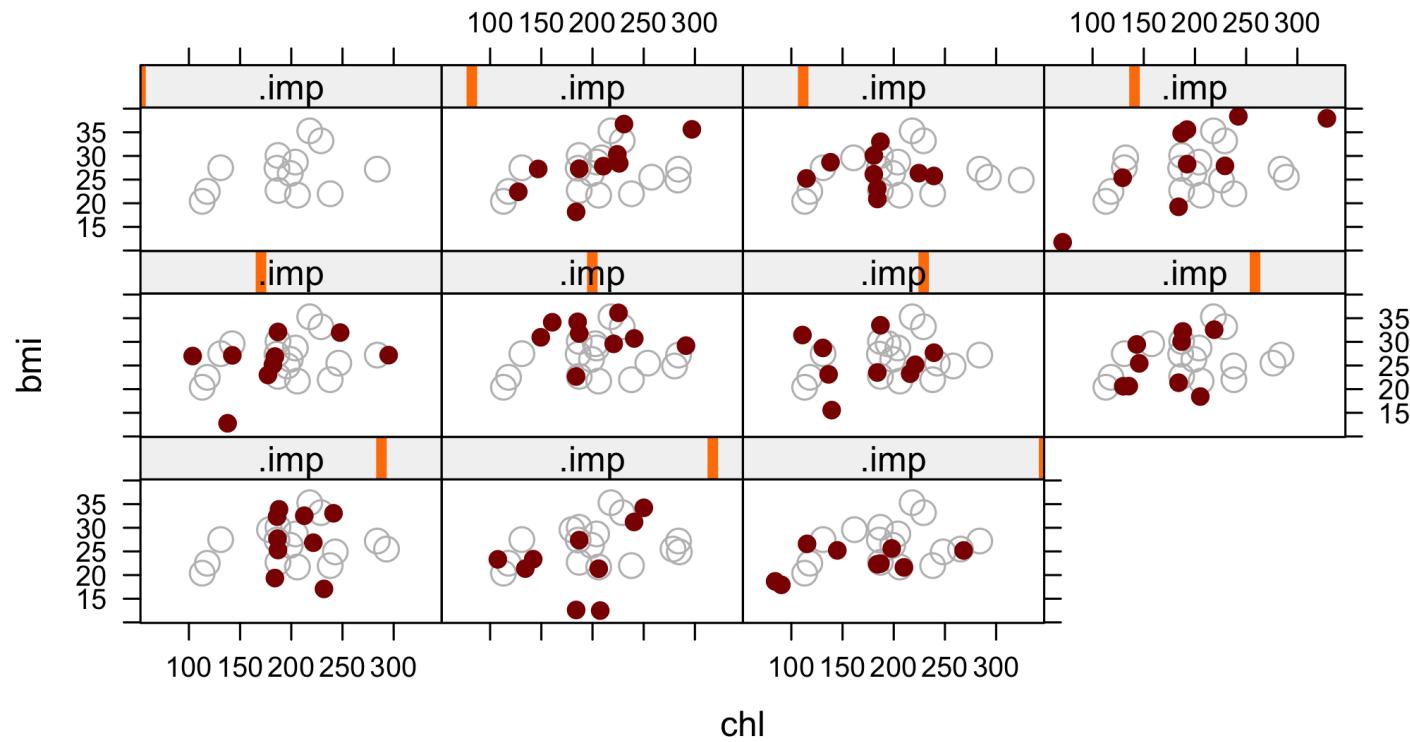


Going forward, let's focus only on imputations using `norm`.

ILLUSTRATION

Scatterplot of `chl` and `bmi` for each imputed dataset. Here we can see why we should not use single imputations.

```
xyplot(nhanes2_imp, bmi ~ chl | .imp, pch=c(1,20), cex = 1.4, col=c("grey", "darkred"))
```



ILLUSTRATION

To detect interesting differences in distribution between observed and imputed data, use the **densityplot** function.

```
densityplot(nhanes2_imp)
```

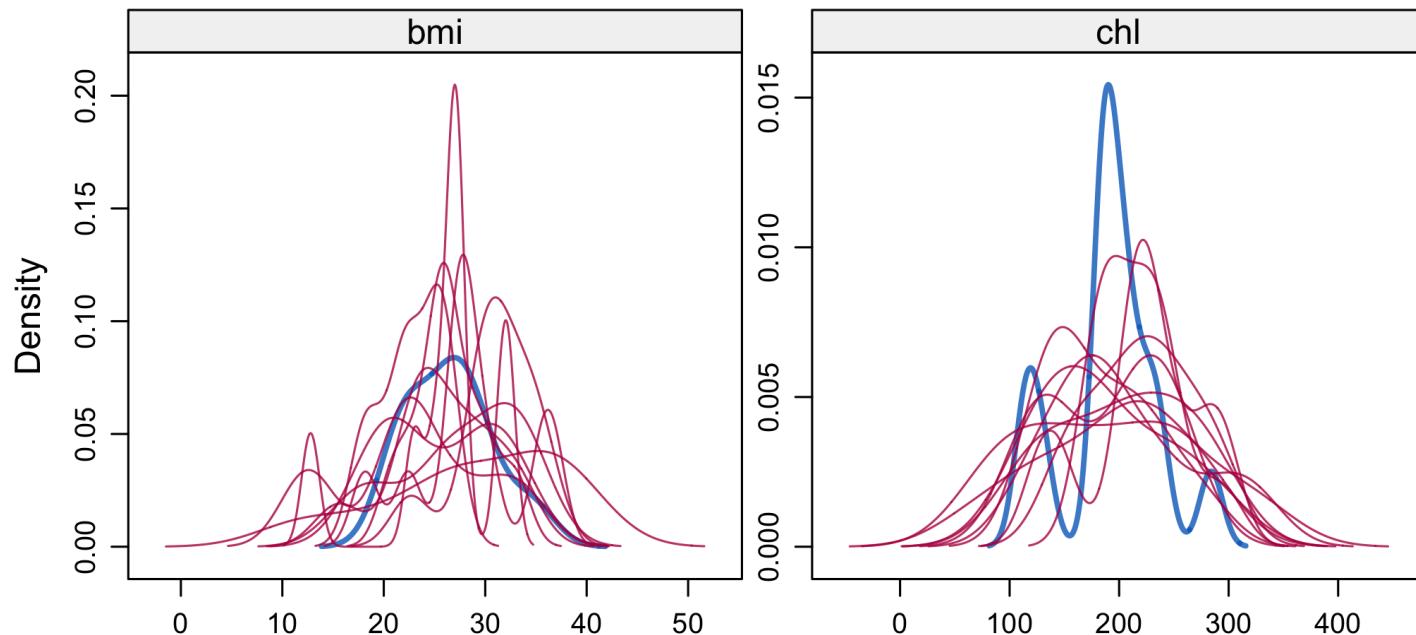


ILLUSTRATION: USING A SINGLE DATASET

For model specification, i.e., transformations, either look at the complete cases or use one of the completed datasets. For example, to use the first dataset in a regression of `bmi` on `age`, `hyp` and `chl`, use

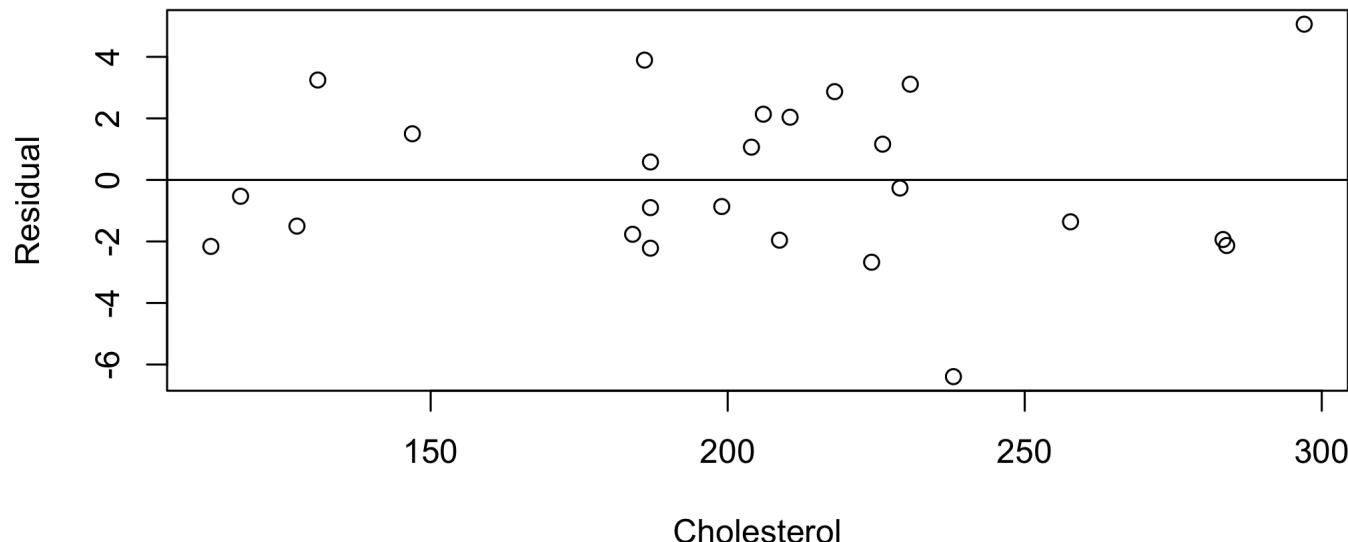
```
bmiregd1 <- lm(bmi~age+hyp+chl, data = d1)
summary(bmiregd1)

##
## Call:
## lm(formula = bmi ~ age + hyp + chl, data = d1)
##
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -6.3936 -1.9368 -0.5314  2.0384  5.0614 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 11.94077  2.74697  4.347  0.000313 ***
## age40-59    -5.91646  1.50677 -3.927  0.000835 ***
## age60-99    -11.73873  2.13243 -5.505 2.18e-05 ***
## hypyes      2.43724  1.71224  1.423  0.170027    
## chl         0.09399  0.01483  6.338 3.48e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.845 on 20 degrees of freedom
## Multiple R-squared:  0.7062,    Adjusted R-squared:  0.6474 
## F-statistic: 12.02 on 4 and 20 DF,  p-value: 3.866e-05
```

ILLUSTRATION: USING A SINGLE DATASET

- To check residuals, you can examine the fit of the model in one or more completed datasets
- Any transformations will have to apply to all the datasets, so don't be too dataset-specific in your checks.

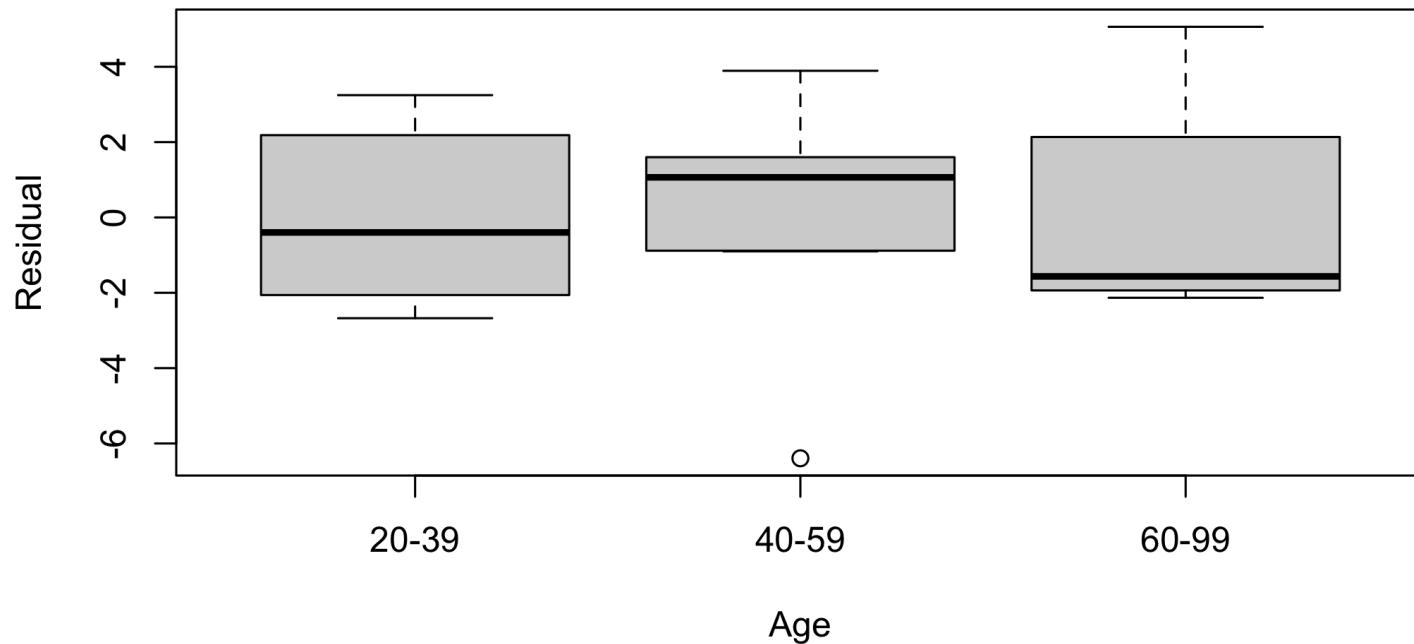
```
plot(bmiregd1$residual,x=d1$chl,xlab="Cholesterol",ylab="Residual"); abline(0,0)
```



Looks good!

ILLUSTRATION: USING A SINGLE DATASET

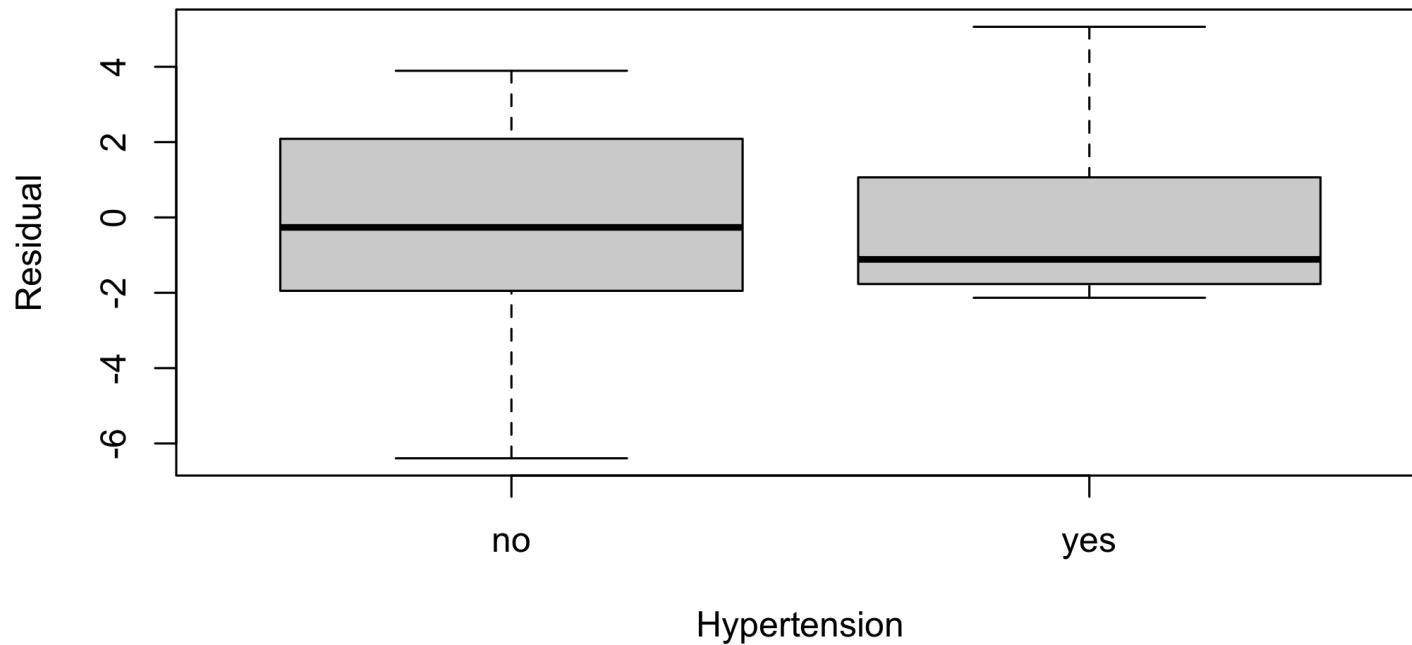
```
boxplot(bmiregd1$residual ~ d1$age, xlab = "Age", ylab = "Residual")
```



Pretty reasonable especially given the size of the dataset.

ILLUSTRATION: USING A SINGLE DATASET

```
boxplot(bmiregd1$residual ~ d1$hyp, xlab = "Hypertension", ylab = "Residual")
```



- Good idea to repeat for more than one completed dataset.
- If you decide transformations are needed, you might reconsider the imputation models too and fit them with transformed values.

ILLUSTRATION: USING ALL M DATASETS

```
bmireg_imp <- with(data=nhanes2_imp, lm(bmi~age+hyp+chl))  
#results for second dataset  
bmireg_imp[[4]][[2]]
```

```
##  
## Call:  
## lm(formula = bmi ~ age + hyp + chl)  
##  
## Coefficients:  
## (Intercept)    age40-59    age60-99    hypyes      chl  
##     18.44372    -6.53144   -10.51157    1.41484    0.06152
```

```
#results for fifth dataset  
bmireg_imp[[4]][[5]]
```

```
##  
## Call:  
## lm(formula = bmi ~ age + hyp + chl)  
##  
## Coefficients:  
## (Intercept)    age40-59    age60-99    hypyes      chl  
##     18.17017    -7.50997   -11.66474    2.31496    0.07016
```

ILLUSTRATION: USING ALL M DATASETS

Now to get the multiple imputation inferences based on the Rubin (1987) combining rules

```
bmireg <- pool(bmireg_imp)
summary(bmireg)

##           term    estimate  std.error  statistic      df   p.value
## 1 (Intercept) 16.03269801 4.37392771 3.6655151 6.317986 0.009594146
## 2 age40-59    -6.57207247 1.97111257 -3.3341944 11.653096 0.006179856
## 3 age60-99   -10.83536538 2.90742365 -3.7267928  7.366405 0.006740526
## 4 hypyes     2.04370148 2.35795119  0.8667276  8.976479 0.408661074
## 5 chl       0.07394739 0.02428787  3.0446227  6.597552 0.020136806
```

ILLUSTRATION: USING ALL M DATASETS

- You can still do a nested F test (well, technically a test that is asymptotically equivalent to a nested F test) for the multiply-imputed dataset using the `pool.compare` function.
- For example, suppose we want to see if age is a useful predictor, then

```
bmireg_imp <- with(data=nhanes2_imp, lm(bmi~hyp+chl+age))
bmireg_impoage <- with(data=nhanes2_imp, lm(bmi~hyp+chl))
#type "pool.compare(bmireg_imp, bmireg_impoage)" to see full results
pool.compare(bmireg_imp, bmireg_impoage)[c(9:12,18)]
```

```
## $qbar1
## (Intercept)      hypyes        chl    age40-59    age60-99
## 16.03269801   2.04370148   0.07394739 -6.57207247 -10.83536538
##
## $qbar0
## (Intercept)      hypyes        chl
## 20.78380117 -0.88512898  0.03078513
##
## $ubar1
## [1] 8.4661183053 3.2824280668 0.0002711616 2.7734500763 4.2665578296
##
## $ubar0
## [1] 1.738379e+01 6.590985e+00 4.824345e-04
##
## $pvalue
## [,1]
## [1,] 1.654266e-05
```

ILLUSTRATION: USING ALL M DATASETS

You also can fit logistic regressions. For example to predict hypertension from all the other variables, do

```
hyplogreg_imp <- with(data=nhanes2_imp, glm(hyp~bmi+chl+age, family = binomial))

## Warning: glm.fit: algorithm did not converge

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

## Warning: glm.fit: algorithm did not converge

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

## Warning: glm.fit: algorithm did not converge

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

This turns out to be problematic here because we have some logistic regressions with perfect predictions.

ILLUSTRATION: USING ALL M DATASETS

```
hyplogreg <- pool(hyplogreg_imp)
summary(hyplogreg)
```

```
##           term   estimate  std.error   statistic      df   p.value
## 1 (Intercept) -5042.073708 788097.633 -6.397778e-03 18.25726 0.9949647
## 2       bmi     90.415352 13854.295  6.526161e-03 18.25698 0.9948637
## 3       chl      8.402631 1359.481  6.180766e-03 18.25763 0.9951355
## 4 age40-59    996.882034 169469.486  5.882369e-03 18.25787 0.9953704
## 5 age60-99    503.458547 8664479.986  5.810603e-05 18.25904 0.9999543
```

We do not have enough data to do a meaningful logistic regression here, unless we drop age as a predictor, but the command structure is fine!

MODIFYING PREDICTORS IN THE IMPUTATION MODELS

Going back to the imputed datasets, which variables does `mice()` use as predictors for imputation of each incomplete variable?

```
nhanes2_imp$predictorMatrix
```

```
##      age bmi hyp chl
## age    0   1   1   1
## bmi    1   0   1   1
## hyp    1   1   0   1
## chl    1   1   1   0
```

MODIFYING PREDICTORS IN THE IMPUTATION MODELS

We can choose to exclude variables from any of the imputation models. For example, suppose we think that `hyp` should not predict `bmi`. Then,

```
pred <- nhanes2_imp$predictorMatrix  
pred["bmi", "hyp"] <- 0  
pred
```

```
##      age bmi hyp chl  
## age  0   1   1   1  
## bmi  1   0   0   1  
## hyp  1   1   0   1  
## chl  1   1   1   0
```

```
mice(nhanes2, m=10, defaultMethod=c("norm", "logreg", "polyreg", "polr"), predictorMatrix=pred)
```

WHAT'S NEXT?

MOVE ON TO THE READINGS FOR THE NEXT MODULE!